NEUTRINO MAJORANA

S. M. Bilenky

Joint Institute for Nuclear Research, Dubna, R-141980, Russia, and SISSA,via Beirut 2-4, I-34014 Trieste, Italy.

Abstract

The Majorana paper "Symmetrical theory of the electron and positron" is briefly reviewed. The present status of Majorana neutrinos is discussed.

1 Introduction

This year we celebrate the 100-th anniversary of the birth of Ettore Majorana, one of the greatest physicist of the XX century. E. Majorana was a very critical person. Especially in the last years of his short life, he did not like to publish his results. He published his most important paper "Symmetrical theory of the electron and positron" [1] in 1937, probably, because of the competition for the chair in theoretical physics at the Palermo University (see [2]). Fermi, Amaldi and other participants of the Fermi group convinced Majorana to take part in the competition. His previous paper (on nuclear forces) was published in 1933 when he was visiting Germany (Heisenberg convinced him at that time to publish the paper).

I will discuss here briefly the content of the Majorana paper. Majorana was not satisfied with the existing at that time theory of electrons and positrons in which positrons were considered as holes in the Dirac sea of the states of electrons with negative energies. He wanted to formulate the symmetrical theory in which there is no notion of states with negative energies.

Let us consider a complex spinor field $\psi(x)$ which satisfies the Dirac equation

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\,\psi(x) = 0. \tag{1}$$

The conjugated field

$$\psi^{c}(x) = C\bar{\psi}^{T}(x) \tag{2}$$

obviously also satisfies the Dirac equation

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\,\psi^{c}(x) = 0. \tag{3}$$

Here C is the matrix of the charge conjugation which satisfies the conditions

$$C\gamma_\alpha^T C^{-1} = -\gamma_\alpha; \quad C^T = -C. \tag{4}$$

Let us present the field $\psi(x)$ in the form

$$\psi(x) = \frac{1}{\sqrt{2}}\chi_1 + i\frac{1}{\sqrt{2}}\chi_2, \tag{5}$$

where

$$\chi_1(x) = \frac{\psi(x) + \psi^c(x)}{\sqrt{2}}; \ \chi_2(x) = \frac{\psi(x) - \psi^c(x)}{\sqrt{2}i}. \tag{6}$$

The fields $\chi_{1,2}(x)$ satisfy the Dirac equation

$$i\gamma^\alpha\partial_\alpha - m)\chi_{1,2}(x) = 0 \tag{7}$$

and *additional conditions*

$$\chi_{1,2}^c(x) = \chi_{1,2}(x). \tag{8}$$

Majorana used the representation in which γ^α are imaginary matrices (Majorana representation). In this representation $\psi^c(x) = \psi^*(x)$ and $\chi_1(x)$ and $\chi_2(x)$ are real and imaginary parts of the field $\psi(x)$. He considered first the fields $\chi_1(x)$ and $\chi_2(x)$ and appling Jordan-Wigner quantization method he constructed the quantum field theory of such fields. Taking into account (4) and (8) it is easy to show that there are no electromagnetic currents for $\chi_{1,2}(x)$:

$$j_i^\alpha(x) = \bar\chi_i(x)\gamma^\alpha\chi_i(x) = -\chi_i^T(x)(\gamma^\alpha)^T\bar\chi_i(x)^T = -\bar\chi_i(x)\gamma^\alpha\chi_i(x) = 0; \ (i = 1, 2) \tag{9}$$

Therefore, $\chi_{1,2}(x)$ are fields of particles with electric charge equal to zero.

For the operator of the energy and momentum Majorana obtained the expression

$$P_i^\alpha = \int\sum_r p^\alpha a_r^\dagger(p)a_r(p)d^3p, \tag{10}$$

where operators $a_r(p)$ and $a_r^\dagger(p)$ satisfy usual anticommutation relations.

Thus, $a_r^\dagger\,(p)(a_r(p))$ is the operator of the creation (absorption) of particle with momentum p and helicity r. There are no states with negative energies and quanta of the fields $\chi_{1,2}(x)$ are neutral particles which are identical to their antiparticles.

Considering complex field $\psi(x)$, presented in the form (5), Majorana came to symmetrical formulation of the theory of particles and antiparticles with the operators of the total momentum and total charge given by

$$P^{\alpha} = \int \sum_{r} p^{\alpha} [c_r^{\dagger}(p) c_r(p) + d_r^{\dagger}(p)\ d_r(p)] d^3p \tag{11}$$

$$Q = e \int \sum_{r} [c_r^{\dagger}(p) c_r(p) - d_r^{\dagger}(p)\ d_r(p)] d^3p \tag{12}$$

Here $c_r^{\dagger}\ (p)(c_r(p))$ is the operator of the creation (absorption) of particle with charge e, momentum p and helicity r and $d_r^{\dagger}\ (p)(d_r(p))$ is the operator of the creation (absorption) of antiparticle with with charge $-e$, momentum p and helicity r. Majorana wrote in the paper "A generalization of Jordan-Wigner quantization method allows not only to give symmetrical form to the electron-positron theory but also to construct an essentially new theory for particles without electric charge (neutrons and hypothetical neutrinos)". And further in the paper: "Although it is perhaps not possible now to ask experiment to choose between the new theory and that in which the Dirac equations are simply extended to neutral particles, one should keep in mind that the new theory is introducing in the unexplored field a smaller number of hypothetical entities"

Soon after the Majorana paper appeared Racah [3] proposed a method which could allow to test whether neutrino is Majorana or Dirac particle. The so-called Racah chain of reactions

$$(A, Z) \to (A, Z+1) + e^- + \nu, \quad \nu + (A', Z') \to (A', Z'+1) + e^- \tag{13}$$

is allowed in the case of the Majorana neutrino and is forbidden in the case of the Dirac neutrino. Of course in 1937 Racah could not know that even in the case of the Majorana neutrino the chain (13) is strongly suppressed (see later).

In 1938 Furry [4] for the first time considered neutrinoless double β-decay of nuclei

$$(A, Z) \to (A, Z+2) + e^- + e^- \tag{14}$$

induced by Racah chain with virtual neutrinos. As we will discuss later the investigation of this process is the major way to probe the nature of neutrinos.

2 Neutrino Majorana; basics

2.1 Interaction Lagrangian

Existing weak interaction data are perfectly described by the Standard Model. The Standard Lagrangians of CC and NC interactions of neutrinos with other particles are given by

$$\mathcal{L}_I^{\mathrm{CC}} = -\frac{g}{2\sqrt{2}}\, j_\alpha^{\mathrm{CC}}\, W^\alpha + \mathrm{h.c.}; \quad \mathcal{L}_I^{\mathrm{NC}} = -\frac{g}{2\cos\theta_W}\, j_\alpha^{\mathrm{NC}}\, Z^\alpha. \tag{15}$$

Here g is the $SU(2)$ gauge coupling, θ_W is the weak angle and

$$j_\alpha^{\mathrm{CC}} = 2\sum_{l=e,\mu,\tau} \bar\nu_{lL}\gamma_\alpha l_L; \quad j_\alpha^{\mathrm{NC}} = \sum_{l=e,\mu,\tau} \bar\nu_{lL}\gamma_\alpha \nu_{lL} \tag{16}$$

are charged and neutral currents.

After the discovery of the neutrino oscillations [5, 6, 7, 8, 9, 10, 11, 12] we know that neutrino masses are different from zero. Thus, in addition to the interaction Lagrangian (and kinetic term) *a neutrino mass term enter into the total Lagrangian*.

Nature of neutrinos with definite masses are determined by the type of neutrino mass term. There are two possible mass terms for neutrinos, particles with equal to zero electric charges (see reviews [13, 14]): I. Majorana mass term; II. Dirac mass term. We will briefly consider first the Dirac mass term.

2.2 Dirac mass term

The Dirac mass term has the form

$$\mathcal{L}^{\mathrm{D}} = -\sum_{l',l} \bar\nu_{l'L}\,(M_{\mathrm{D}})_{l'l}\nu_{lL} + \mathrm{h.c.}, \tag{17}$$

where M_{D} is a complex 3×3 matrix. After the standard diagonalization of the matrix M_{D} for the mass term we have

$$\mathcal{L}^{\mathrm{D}} = -\sum_{i=1}^{3} m_i \bar\nu_i \nu_i, \tag{18}$$

where $\nu_i(x)$ is the field of neutrino with mass m_i. Flavor fields $\nu_{lL}(x)$ is connected with left-handed fields $\nu_{iL}(x)$ by the mixing relation

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\ \nu_{iL}(x), \tag{19}$$

where U is unitary PMNS mixing matrix.

In the case of the Dirac mass term (17) the total Lagrangian is invariant under global gauge transformations

$$\nu'_{lL}(x) = e^{i\alpha}\ \nu_{lL}(x),\ \nu'_{lR}(x) = e^{i\alpha}\ \nu_{lR}(x),\ l'(x) = e^{i\alpha}\ l(x),\ q'(x) = q(x), \tag{20}$$

where α is arbitrary constant phase.

Invariance under the transformations (20) means that the total lepton number

$$L = L_e + L_\mu + L_\tau \tag{21}$$

is conserved and that $\nu_i(x)$ is four component field of neutrinos ($L(\nu_i) = 1$) and antineutrinos ($L(\bar{\nu}_i) = -1$).

2.3 Majorana mass term

We will consider now the Majorana mass term. Let us introduce the column of the left-handed fields

$$n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{s_1 L} \\ \vdots \end{pmatrix}. \tag{22}$$

We assumed that to addition to flavor fields ν_{lL} sterile fields $\nu_{s_i L}$ can enter into the mass term. From (4) it follows that

$$(n_L)^c = C\ \bar{n}_L^T, \tag{23}$$

is the column of right-handed fields.

Majorana mass term is a Lorenz-invariant product of left-handed components $\bar{n}_{\alpha' L}$ and right-handed components $(n_{\alpha L})^c$:

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\,\bar{n}_L\, M^{\mathrm{M}} (n_L)^c + \mathrm{h.c.}, \tag{24}$$

where M^{M} is a symmetrical $(3+n_s)\times(3+n_s)$ matrix (n_s is the number of sterile fields).

A symmetrical matrix can be diagonalized with the help of an unitary matrix:

$$M^{\mathrm{M}} = U\, m\, U^{T}, \tag{25}$$

where $U^{\dagger}\, U = 1$ and $m_{ik} = m_i\, \delta_{ik};\quad m_i > 0$.

From (24) and (25) for the mass term we have

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\; \bar{\nu}_L\; m\; (\nu_L)^c + \mathrm{h.c.}, \tag{26}$$

where

$$\nu_L = U^{\dagger}\; n_L. \tag{27}$$

From (26) for the mass term we finally obtain

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\, \bar{\nu}\; m\; \nu = -\frac{1}{2}\, \sum_{i=1}^{3+n_s} m_i\, \bar{\nu}_i \nu_i. \tag{28}$$

Here

$$\nu = \nu_L + (\nu_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \vdots \end{pmatrix}. \tag{29}$$

We conclude from (28) and (29) that the field $\nu_i(x)$ is the field of neutrino with mass m_i which satisfy the Majorana condition

$$\nu_i^c(x) = \nu_i(x). \tag{30}$$

From (22) and (27) for the neutrino mixing we find

$$\nu_{lL} = \sum_{i=1}^{3+n_s} U_{li}\nu_{iL};\quad \nu_{s_k L} = \sum_{i=1}^{3+n_s} U_{s_k i}\nu_{iL}. \tag{31}$$

The Majorana condition (30) is equivalent to the relation

$$\nu_{iR}(x) = (\nu_{iL}(x))^c \tag{32}$$

Therefore, right-handed and left-handed components of the Majorana fields are connected by the relation (32). Let us stress that in the case of the Dirac field right-handed and left-handed components are independent.

From Majorana condition (30) for the field $\nu_i(x)$ we obtain

$$\nu_i(x) = \int \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} \left(e^{-ipx}\, u^r(p)\, a_r^i(p) + e^{ipx}\, v^r(p)\, a_r^{i\dagger}(p)\right) d^3p. \qquad (33)$$

Here $a_r^i(p)$ and $a_r^{i\dagger}(p)$ are operators of absorption and creation of neutrino with momentum p, helicity r and mass m_i. Thus, *Majorana neutrinos and antineutrinos are identical particles.*

There exist at present strong arguments in favor of Majorana nature of massive neutrinos. These arguments are based on the fact that neutrino masses are much smaller than masses of quarks and leptons. Absolute values of neutrino masses at present are unknown. From the data of the tritium experiments it was found [15]

$$m_i \leq 2.3 \text{ eV} \qquad (34)$$

From different analysis of the cosmological data for the sum of neutrino masses the bounds in the range

$$\sum_i m_i \leq (0.4 - 1.7) \text{ eV} \qquad (35)$$

were inferred [16]. These bounds are many orders of magnitude smaller than masses of quarks and leptons.

The most natural explanation of the smallness of neutrino masses is based on the assumption that the total lepton number is violated by a right-handed Majorana mass term at a large scale (the famous see-saw mechanism [17] of neutrino mass generation).

Let us assume that the Dirac mass term is generated by the Standard Higgs mechansm. We can expect that eigenvalues of the matrix $M_{\rm D}$ are of the order of quark or lepton masses. Taking into account that neutrino masses are much smaller than masses of quarks and leptons we will assume that lepton number violating right-handed Majorana mass term

$$\mathcal{L}^{\rm M_R} = -\sum_{l',l} \overline{(\nu_{l'R})^c}\, (M_{\rm R})_{l'l} \nu_{lR} + \text{h.c.}, \qquad (36)$$

with eigenvalues of $M_{\rm R}$ which are much larger than masses of leptons and quarks, is generated by some new mechanism. We assume also that the left-handed Majorana mass term is equal to zero. For the Majorana mass metrix we have in this case

$$M^{\rm M+D} = \left(\begin{array}{cc} 0 & M_{\rm D}^{T} \\ M_{\rm D} & M_{R} \end{array} \right), \tag{37}$$

The matrix $M^{\rm M+D}$ can be presented in block-diagonal form

$$U^{T}\, M^{\rm M+D}\, U \simeq \left(\begin{array}{cc} m_{\nu} & 0 \\ 0 & M_{R} \end{array} \right), \tag{38}$$

where

$$m_{\nu} = -M_{\rm D}\, M_{R}^{-1}\, M_{\rm D}^{T} \tag{39}$$

is the Majorana neutrino mass matrix.

From (39) it follows that neutrino masses are much smaller than masses of quarks and leptons. Values of neutrino masses and neutrino mixing angles can be specified only in the framework of concrete models.

If see-saw mechanism is responsible for Majorana neutrino mass generation in this case heavy Majorana particles, see-saw partners of light Majorana neutrinos, must exist. CP-violating decays of these particles in the early Universe is considered as a plausible source of the barion asymmetry of the Universe (see review [18]).

2.4 Majorana mixing matrix

($|m_{\beta\beta}| \leq 1$ eV)

An unitary $n \times n$ matrix U is characterized by $\frac{n(n-1)}{2}$ angles and $\frac{n(n+1)}{2}$ phases. The matrix U can be presented in the form

$$U = S^{\dagger}(\beta)\; U^{0}\; S(\alpha) \tag{40}$$

where

$$S_{l'l}(\beta) = e^{i\beta_{l}}\, \delta_{l'l}; \quad S_{ik}(\alpha) = e^{i\alpha_{i}}\, \delta_{ik}. \tag{41}$$

A common phase is unobservable. Thus one phase in $S(\alpha)$ and $S(\beta)$ can be put equal to zero. We will choose $\alpha_{n} = 0$.

Let us consider first the Dirac case. For the CC we have $2\sum_{l} \bar{l}_{L}\gamma_{\alpha}\nu_{lL} = 2\sum_{l,i} \bar{l}_{L}\gamma_{\alpha}U_{li}\nu_{iL}$. Phases of Dirac fields are unmeasurable quantities. Thus, phase factors $e^{i\beta_l}$ and $e^{i\alpha_i}$ can be included into the fields $l(x)$ and $\nu_i(x)$, respectively, Therefore, the Dirac mixing matrix is given by

$$U^D = U^0 \tag{42}$$

This matrix is characterized by $\frac{n(n+1)}{2} - (2n-1) = \frac{(n-1)(n-2)}{2}$ physical phases and $\frac{n(n-1)}{2}$ angles. In $n = 3$ case the Dirac mixing matrix is characterized by three angles and one phase.

In the case of the Majorana neutrinos only phase factors $e^{i\beta_l}$ can be absorbed by the Dirac fields $l(x)$. Majorana mixing matrix has the form [19, 20]

$$U^M = U^0\, S(\alpha) \tag{43}$$

It is characterized by $\frac{n(n-1)}{2}$ angles and $\frac{n(n+1)}{2} - n = \frac{n(n-1)}{2}$ physical phases. In $n = 3$ case the Majorana mixing matrix is characterized by three angles and three phases.

2.5 Neutrinoless double β decay

The investigation of the neutrinoless double β decay

$$(A, Z) \to (A, Z + 2) + e^- + e^- \tag{44}$$

of some even-even nuclei is the most sensitive method which could allow to reveal the Majorana nature of neutrinos with definite masses. (see reviews [21]). In this subsection we will briefly discuss this process. We will start with the following remarks.

1. The investigation of neutrino oscillations in vacuum or in matter does not allow to reveal the nature of ν_i [19, 22]. In fact, the probability of the transition $\nu_l \to \nu_{l'}$ in vacuum is given by (see [14])

   $$\mathrm{P}(\nu_l \to \nu_{l'}) = \mid \sum_i U_{l'i}\ e^{-i\Delta m^2_{1i}\ \frac{L}{2E}}\ U^*_{li} \mid^2, \tag{45}$$

   where L is the distance between neutrino production and neutrino detection points, E is neutrino energy and $\Delta m^2_{1i} = m^2_i - m^2_1$. From (43) and (45) it is obvious that additional Majorana phases α_i drop out from the expression for the transition probability. Thus, we have

   $$\mathrm{P}^M(\nu_l \to \nu_{l'}) = \mathrm{P}^D(\nu_l \to \nu_{l'}) \tag{46}$$

Similarly it can be shown [22] that the study of neutrino transitions in matter also does not allow to reveal the nature of massive neutrinos.

2. For the SM weak interaction, theories with massless Dirac and Majorana neutrinos are equivalent [23].

   We have stressed before that the major difference between Dirac and Majorana fields is connected with right-handed components: in the Dirac case right-handed and left-handed components are independent, while in the Majorana case right-handed and left-handed components are connected by the relations (32). If $m_i = 0$, the right-handed fields do not enter into the Lagrangian. Hence, there is no possibility to distinguish Dirac and Majorana neutrinos in this case.

   For illustration of the equivalence theorem let us consider Racah chain Eq.(13). From (15) and (16) it is obvious that in the first reaction together e^- right-handed neutrino is produced. However, in order to produce e^- in the second reaction of the chain left-handed neutrino must be absorbed. Thus, for massless neutrino the Racah chain is forbidden: neutrino helicity plays a role of the lepton number.

3. Let us consider the Racah chain in the case of neutrinos with different from zero masses. In the matrix elements of the process of production of neutrino with momentum p, mass m_i and helicity r enter the spinor $\frac{1-\gamma_5}{2}v^r(p)$. Taking into account linear in $\frac{m_i}{2E}$ terms, we have

$$\frac{1-\gamma_5}{2}\; v^r(p) = \frac{1+r}{2}\; v^r(p) + r\frac{m_i}{2E}\gamma^0 v^r(p), \tag{47}$$

   where E is neutrino energy. In the matrix elements of the process of absorption of neutrino with momentum p, mass m_i and helicity r enter the spinor

$$\frac{1-\gamma_5}{2}\; u^r(p) = \frac{1-r}{2}\; u^r(p) + r\frac{m_i}{2E}\gamma^0 u^r(p). \tag{48}$$

   From (47) it follows that in the neutrino-production process mainly right-handed neutrinos are produced. From (48) we see that in the cross section of the absorption of such neutrinos in the second process of the chain small factors $(\frac{m_i}{E})^2$ enter. The probability of the production of the left-handed neutrinos, which have "large" weak absorption cross section, is suppressed by the factor $(\frac{m_i}{E})^2$. Therefore, for massive Majorana neutrinos the Racah chain is suppressed by the helicity suppression factor $(\frac{m_3}{E})^2 \lesssim 10^{-12}$. (in neutrino processes $E \gtrsim$ MeV). We conclude that it is not possible in foreseeable future to reveal neutrino nature in neutrino experiments of the Racah type.

Possibilities to use large targets (in present-day experiments tens of kg, in future experiments about 1 ton and may be more), to reach small background and high energy resolution make experiments on the search for $0\nu\beta\beta$ decay an unique source of information about the nature of massive neutrinos ν_i Neutrinoless double β-decay (44) is the second order in the Fermi constant process with virtual neutrinos. For mixed neutrino field

$$\nu_{eL} = \sum_i U_{ei} \nu_{iL} \tag{49}$$

neutrino propagator is given by

$$\langle 0 \mid T(\nu_{eL}(x_1)\, \nu^T_{eL}(x_2)) \mid 0 \rangle \simeq m_{\beta\beta}\, \frac{-i}{(2\,\pi)^4}\, \frac{1-\gamma_5}{2}\, C \int e^{-ip(x_1-x_2)} \frac{1}{p^2}\, d^4p, \tag{50}$$

where

$$m_{\beta\beta} = \sum_i U^2_{ei}\, m_i \tag{51}$$

is *effective Majorana mass.*

For half-life of $0\nu\beta\beta$-decay the following general expression can be obtained[21]

$$\frac{1}{T^{0\,\nu}_{1/2}(A,Z)} = |m_{\beta\beta}|^2\, |M(A,Z)|^2\, G^{0\,\nu}(E_0,Z), \tag{52}$$

where $M(A,Z)$ is nuclear matrix element and $G^{0\,\nu}(E_0,Z)$ is known phase-space factor (E_0 is the energy release). Nuclear matrix elements are determined only by nuclear properties and strong interaction and does not depend on neutrino masses. The calculation of nuclear matrix elements $|M(A,Z)|$ is a complicated nuclear problem which we will briefly discuss later on.

There exist at present data of many experiments on the search for $0\nu\beta\beta$-decay (see [24]). The stringent lower bound on the half-time of the neutrinoless double β decay was obtained in the germanium Heidelberg-Moscow experiment[1] [25]

$$T^{0\,\nu}_{1/2}(^{76}\mathrm{Ge}) \geq 1.55 \cdot 10^{25} \mathrm{years} \quad (90\%\mathrm{CL}) \tag{53}$$

[1] An indication in favor of $0\nu\beta\beta$-decay of ^{76}Ge, found in [26], is going to be checked by the GERDA experiment started at Gran Sasso [27].

Taking into account different calculations of the nuclear matrix element, from this result for the effective Majorana mass upper bounds in the following range

$$|m_{\beta\beta}| \leq (0.3 - 1.2)\ \text{eV}. \quad (\text{Heidelberg} - \text{Moscow}) \tag{54}$$

can be inferred.

The same sensitivity to the effective Majorana mass was reached in the recent cryogenic experiment CUORICINO [28]. For half-life of ^{130}Te in this experiment the following lower bound was found

$$T^{0\,\nu}_{1/2}(^{130}\text{Te}) \geq 1.8 \cdot 10^{24}\,\text{years} \quad (90\%\text{CL}) \tag{55}$$

From this result it was obtained

$$|m_{\beta\beta}| \leq (0.2 - 1.1)\ \text{eV}. \quad (\text{CUORICINO}) \tag{56}$$

Several future experiments on the search for $0\,\nu\beta\,\beta$-decay (CUORE, MAJORANA, EXO, SUPER-NEMO and others) are in preparation at present.[29]. The aim of these future experiments is to reach sensitivity

$$|m_{\beta\beta}| \simeq \text{a few } 10^{-2}\ \text{eV}. \tag{57}$$

2.6 The effective Majorana mass

The observation of $0\nu\beta\beta$-decay would be a direct proof that massive neutrinos ν_i are Majorana particles. As we will see in this subsection the determination of the effective Majorana mass $|m_{\beta\beta}|$ would allow to obtain an important information on the character of neutrino mass spectrum and the lightest neutrino mass.

All neutrino oscillation data (with the exception of the data of the LSND experiment [30]) are described by the three-neutrino mixing[2] The three-neutrino transition probabilities depend on six parameters: two neutrino mass-squared differences Δm^2_{12} and Δm^2_{23}, three mixing angles θ_{12}, θ_{23} and θ_{13} and CP phase δ.

From analysis of the Super Kamiokande atmospheric data the following 90 % CL ranges were obtained [5]

$$1.5 \cdot 10^{-3} \leq \Delta m^2_{23} \leq 3.4 \cdot 10^{-3}\text{eV}^2; \quad \sin^2 2\theta_{23} > 0.92. \tag{58}$$

[2]The LSND indication in favor of $\bar{\nu}_\mu \to \bar{\nu}_e$ are going to be checked by the MiniBooNE experiment [31].

From the global analysis of solar and KamLAND data it was found [6]

$$\Delta m^2_{12} = 8.0^{+0.6}_{-0.4}\ 10^{-5}\ \mathrm{eV}^2; \quad \tan^2\theta_{12} = 0.45^{+0.09}_{-0.07}. \tag{59}$$

From the result of the reactor CHOOZ experiment [32] for the parameter $\sin^2\theta_{13}$ the following upper bound was obtained

$$\sin^2\theta_{13} \leq 5 \cdot 10^{-2} \tag{60}$$

The current neutrino oscillation experiments do not allow to distinguish two types of neutrino mass spectra possible in the case of the three-neutrino mixing

1. Normal spectrum

$$m_1 < m_2 < m_3;\ \Delta m^2_{12} \ll \Delta m^2_{23} \tag{61}$$

2. Inverted spectrum

$$m_3 < m_1 < m_2;\ \Delta m^2_{12} \ll |\Delta m^2_{13}| \tag{62}$$

For the lightest neutrino mass $m_0 = m_1(m_3)$ only the upper bounds (34) and (35) are known from the data of the tritium experiments [15] and cosmological data [16].

Effective Majorana mass $m_{\beta\beta}$ strongly depends on the value of the lightest neutrino mass and on the type of the neutrino mass spectrum (see review [33] and references therein). We will consider three standard neutrino mass spectra.

1. Hierarchy of neutrino masses

$$m_1 \ll m_2 \ll m_3 \tag{63}$$

Neglecting the contribution of m_1, for the effective Majorana mass we obtain the following expression

$$|m_{\beta\beta}| \simeq \left| \sin^2\theta_{12}\sqrt{\Delta m^2_{12}} + e^{2i\,\alpha_{23}}\sin^2\theta_{13}\sqrt{\Delta m^2_{23}} \right|, \tag{64}$$

where $\alpha_{23} = \alpha_3 - \alpha_2$ is the difference of the Majorana CP phases.

The first term in Eq.(64) is small because of the smallness of $\sqrt{\Delta m_{12}^2}$. Contribution of the "large" $\sqrt{\Delta m_{23}^2}$ is suppressed by the small factor $\sin^2\theta_{13}$. From (58), (59) and (60) for the upper bound of the effective Majorana mass we find

$$|m_{\beta\beta}| \leq 6.6 \cdot 10^{-3} \text{ eV}. \tag{65}$$

Thus, upper bound of $|m_{\beta\beta}|$ in the case of the neutrino mass hierarchy is smaller that the expected sensitivity of the future experiments on the search for $0\nu\beta\beta$-decay. The value of $|m_{\beta\beta}|$ could be significantly smaller than (65) if cancellation of two terms in (64) takes place.

2. Inverted hierarchy of neutrino masses

$$m_3 \ll m_1 < m_2. \tag{66}$$

For the effective Majorana mass we obtain the following expression

$$|m_{\beta\beta}| \simeq \sqrt{|\Delta m_{13}^2|}\ \ (1 - \sin^2 2\,\theta_{12}\,\sin^2\alpha_{12})^{\frac{1}{2}}, \tag{67}$$

where the only unknown parameter is $\sin^2\alpha_{12}$. From (67) we have the range

$$\cos 2\,\theta_{12}\,\sqrt{|\Delta m_{13}^2|} \leq |m_{\beta\beta}| \leq \sqrt{|\Delta m_{13}^2|} \tag{68}$$

Taking into account (58) and (59) we find that the effective Majorana mass can take the values

$$0.9 \cdot 10^{-2} \leq |m_{\beta\beta}| \leq 5.8\cdot, 10^{-2} \text{ eV} \tag{69}$$

which are in the range of the anticipated sensitivities to $|m_{\beta\beta}|$ of the future experiments on the search for $0\nu\beta\beta$-decay. Thus, next generation of the $0\nu\beta\beta$- experiments can probe the nature of massive neutrinos in the case of the inverted hierarchy of the neutrino masses.

3. Quasi-degenerate neutrino mass spectrum. If the lightest neutrino mass satisfies inequality

$$m_0 \gg \sqrt{|\Delta m_{23}^2|} \tag{70}$$

neutrino mass spectrum is practically degenerate

$$m_1 \simeq m_2 \simeq m_3 \tag{71}$$

The effective Majorana mass is given in this case by

$$|m_{\beta\beta}| \simeq m_0\,(1 - \sin^2 2\,\theta_{12}\,\sin^2 \alpha_{12})^{\frac{1}{2}}, \tag{72}$$

From (70) and (72) we conclude that in the case of the quasi-degenerate spectrum much larger values of the effective Majorana mass are expected than in the case of the inverted hierarchy. Such values can be probed in the ongoing $0\nu\beta\beta$-experiments. Notice that from the observation of the $0\nu\beta\beta$-decay an information about the value of m_0 can be inferred:

$$|m_{\beta\beta}| \leq m_0 \leq 4.4\,|m_{\beta\beta}| \tag{73}$$

Three neutrino mass spectra, we have considered, correspond to different mechanisms of neutrino mass generation (see [34]). Masses of quarks and charged leptons satisfy hierarchy of the type (63). Hierarchy of neutrino masses is a typical feature of GUT models (like SO(10)) in which quarks and leptons are unified. Inverted spectrum and quasi-degenerate spectrum require specific symmetries of the neutrino mass matrix.

In order to determine effective Majorana mass from experimental data nuclear matrix elements (NME) must be known. Two different approaches are used for the calculation of NME (see reviews [35]).: Nuclear Shell Model and Quasiparticle Random Phase Approximation . Different calculations of NME differ by factor 2-3 and more. It is important to find a possibility to test NME calculations. This will be possible if $0\nu\beta\beta$-decay of *different nuclei* is observed. If Majorana neutrino mass mechanism is the dominant mechanism of the decay, the matrix element of the process is factorized in the form of the product of $m_{\beta\beta}$ and NME. Thus, ratio of NME of different nuclei is determined by the ratio of the corresponding half-lives. This can be used as a model independent test of different calculations [36].

3 Conclusion

It was established by the oscillation experiments that neutrino masses are different from zero and flavor neutrinos ν_e, ν_μ and ν_τ are mixed particles. In order to reveal the origin of the small neutrino masses it is crucial to determine the nature of neutrinos with definite masses.

There is no theory of neutrino masses at present. There exist different strategies and models. One of the most natural (and popular) strategy is see-saw. The see-saw mechanism is based on the assumption that the total lepton number L is violated on a large scale $\simeq 10^{-15}$ GeV by a right-handed Majorana mass term. If see-saw mechanism is realized in this case neutrinos with definite masses are Majorana particles.

The search for neutrinoless double β-decay of some even-even nuclei is the most sensitive method of the investigation of the nature of neutrinos with definite mass. The sensitivity to the effective Majorana mass of the future experiments, now under preparation, is planned to be about two order of magnitude better than the sensitivity of the current experiments.

Are massive neutrinos and antineutrinos identical or different ? This problem, which has been put forward by E. Majorana about 70 years ago, is the most fundamental problem of the modern neutrino physics. Without its solution the origin of the small neutrino masses and neutrino mixing can not be revealed.

I would like to acknowledge the Italian program "Rientro dei cervelli" for the support.

References

[1] E. Majorana, Nuovo Cimento, **5** (1937) 171.

[2] B. Pontecorvo, Journal de Physique, **43** (1982) No. 12, C8-221.

[3] G. Racah, Nuovo Cimento, **14** (1937) 322.

[4] W. Furry, Phys. Rev. **56** (1938) 1184.

[5] Super-Kamiokande Collaboration, Y. Ashie et al., Phys. Rev. Lett. **93** (2004) 101801, Y. Ashie *et al.,* Phys.Rev. **D71** (2005) 112005..

[6] SNO Collaboration, Phys.Rev.Lett. **81** (2001) 071301; **89** (2002) 011301 ; **89** (2002) 011302; Phys.Rev. **C72** (2005) 055502

[7] KamLAND Collaboration, T.Araki *et al.*, Phys.Rev.Lett. **94** (2005) 081801; Phys.Rev.Lett. **94** (2005) 081801

[8] K2K Collaboration, E. Aliu *et al.*, Phys.Rev.Lett. **94**(2005) 081802.

[9] B. T. Cleveland *et al.*, Astrophys. J. **496** (1998) 505.

[10] GALLEX-GNO Collaboration, M. Altmann *et al.*, Phys. Lett. **B 490** (2000) 16 ; Nucl.Phys.Proc.Suppl. **91** (2001) 44.

[11] SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Rev. **C 60** (1999) 055801 ; Nucl.Phys.Proc.Suppl. **110** (2002) 315.

[12] Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86** (2001) 5651.

[13] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59**, (1987) 671.

[14] S.M. Bilenky, C. Giunti and W. Grimus. Prog. Part. Nucl. Phys. **43**, (1999) 1.

[15] Ch. Kraus *et al.* Eur. Phys. J. **C40** (2005) 447. V.M. Lobashev *et al.*, Prog. Part. Nucl. Phys. **48** (2002) 123.

[16] M. Tegmark, hep-ph/0503257; O. Elgaroy and O. Lahav, New J. Phys. **7** (2005) 61. S. Hannestad, H. Tu and Y.T.T. Wong, astro-ph/0603019.

[17] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, p. 315, edited by F. van Nieuwenhuizen and D. Freedman, North Holland, Amsterdam, 1979 ; T. Yanagida, Proc. of the *Workshop on Unified Theory and the Baryon Number of the Universe*, KEK, Japan, 1979 ; P. Minkovski, Phys. Lett. **B 67** (1977) 421. S.L. Glashow, Proceedings of Cargese Summer Institute on Quarks and Leptons, Plenum Press, New York, 1980, pp. 687-713. R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[18] W. Buchmuller, R. D. Peccei, T. Yanagida, Ann.Rev.Nucl.Part.Sci. **55** (2005) 311-355,

[19] S.M. Bilenky, J. Hosek and S.T. Petcov, Phys. Lett. **B94** (1980) 495.

[20] J. Schechter and J.W.F. Valle, Phys. Rev. **D23** (1980) 2227; M. Doi *et al.*, Phys. Lett. **B 102** (1981) 323; J. Bernabeu and P. Pascual, Nucl. Phys. **B 228** (1983) 21.

[21] M. Doi, T. Kotani and E. Takasugi, Progr. Theor. Phys. Suppl. **83** (1985) 1; S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59** (1987) 671; J.D. Vergados, Phys. Rep. **361** (2002) 1; S.R. Elliott and P. Vogel, Ann. Rev. Nucl. Part. Sci. **52** (2002) 11.

[22] P. Langacker *et al.*, Nucl. Phys. **B 282** (1987) 589.

[23] C.Ryan and S. Okubo, Nuove Cimento Suppl. **2** (1964) 234; K.M. Case, Phys. Rev. **107** (1957) 307.

[24] I.V. Tretyak and Yu.G. Zdesenko, At. Data Nucl. Data Tables **80** (2002) 83; A.S.Barabash, Physics of Atomic Nuclei, **67** (2004) 438.

[25] Heidelberg-Moscow collaboration, H. V. Klapdor-Kleingrothaus *et al.*, Eur. Phys. J. **A12** (2001) 147; A.M. Bakalyarov *et al.* Phys. Part. Nucl. Lett. 2 (2005) 77; Pisma Fiz. Elem. Chast. Atom. Yadra 2 (2005) 21.

[26] H.V. Klapdor-Kleingrothaus *et al.*, Phys. Lett. B **586**, (2004) 198.

[27] Gerga collaboration I. Abt *et al.*, hep-ex/0404039.

[28] CUORE Collaboration, Arnaboldi, C. *et al.,* Phys. Rev. Lett. 95 (2005) 142501; hep-ex/0501034.

[29] S.R. Elliott and J.Engel, J. Phys. G30 (2004) R183, hep-ph/0405078; C.Aalseth *et al.,* hep-ph/0412300.

[30] LSND Collaboration, A. Aguilar et al., Phys. Rev. D **64**, 112007 (2001). G. Drexlin ,Nucl. Phys. Proc. Suppl. 118, 146 (2003).

[31] MiniBooNE Collaboration, A.A. Aguilar-Arevalo, hep-ex/0408074; Heather L. Ray *et al.*, hep-ex/0411022.

[32] CHOOZ collaboration, M. Apollonio *et al.*, Eur. Phys. J. **C27** (2003) 331;

[33] S. M. Bilenky, J. Phys. **G32** (2006) R127.

[34] G. Altarelli and F.Feruglio, New J. Phys. **6**, 106 (2004) 106;

[35] A. Faessler and F.Šimkovic, J. Phys. **G24** (1998) 2139; J. Suhonen and O. Civitarese, Phys. Rep. **300** (1998) 123.

[36] S.M. Bilenky and J.A. Grifols, Phys. Lett. **B550** (2002) 154; S.M. Bilenky and S.Petcov hep-ph/0405237.